

## KINETIC ANALYSIS OF THE HUMAN KNEE JOINT

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**Abstract.** The pathology of the calcaneal (Achilles) tendon constitutes a serious therapeutical and social problem. Indeed, this tendon is the strongest plantar flexor of the foot that plays an important role in the humangait. Although well known for a long time, no explicit description of the spontaneous subcutaneous rupture of the Achilles tendon can be found in medical or biomechanical literature. So far, neither pathomechanism nor the underlying causes of the tendon's disruption have been fully elucidated. Many authors concentrate mostly on medical and biological aspects of the condition. The commonly held view is that it is the vascular supply to the tendon that plays a crucial role in pathogenesis of the tendon's injuries. In fact, the vasculature a change with time and after the age of 30 is significantly reduced leading to the development of regressive alterations within as well as the decrement of the mechanical strength of the tendon. Obviously, interdisciplinary approach encompassing not only medical and biological but also the broadly taken mechanical viewpoint is needed to more comprehensively describe and explain this phenomenon. In the present paper, kinetic analysis of the knee was employed to define the trajectory of the point of initial insertion of the medial head of gastrocnemius, which was then used to determine the point's route within the motor area extending from the flexion to the full extension of the knee. The obtained data on the trajectory are further utilized to present and define the pathomechanism of the spontaneous rupture of the calcaneal tendon.

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### Introduction

In all the descriptions to date of the patomechanism of the spontaneous disruption of the calcaneal tendon numerous factors have been implicated as

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important for the mechanics of the muscles and bones of the human lower limb during generation of the motion, i.e., supporting the body during walking, running, and jumping [6].

Pathomechanism of the spontaneous rupture of the calcaneal tendon has not been explained predominantly by a mechanical theory according to which a rapid, uncoordinated contraction of the triceps muscle of the calf, especially at the plantar flexion of the foot and with the extended leg, leads to disruption of the tendon. However, this very general theory should be disregarded as it does not consider the cause-effect relationships involved without which descriptions of such injuries are too enigmatic and do not account for the complexity of the phenomenon. Moreover, the theory lacks mathematical background necessary for description of the physical events leading to the subcutaneous rupture of the calcaneal tendon. Hence, interdisciplinary investigations encompassing not only bio-medical but also the broadly understood mechanical approaches are needed in order to broaden our knowledge and provide the more comprehensive description and explanation of this phenomenon.

*Aim of the study:* The aims of the present investigation included the following:

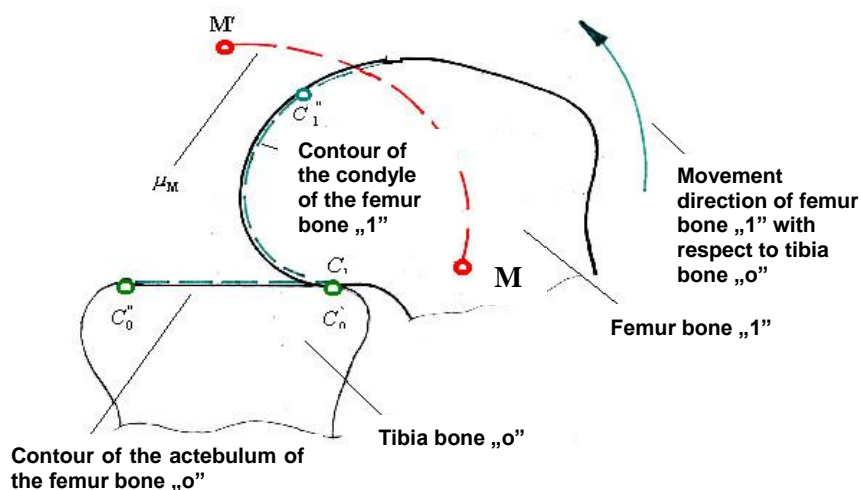
1. Kinetic analysis of the knee joint using a mathematical apparatus in order to determine the coordinates on the trajectory of the point movement of the initial insertion of the medial head of gastrocnemius from the flexion of the femur to its full extension;
2. Calculation of the length of the path covered by the point of the initial insertion of the medial head of gastrocnemius;
3. Assessment of the value of the slide of the medial condyle of the femur against the upper articular surface of the medial condyle of the tibia and its derivatives: the velocity and acceleration during extension of the knee.

## Materials and Methods

In discussions about the pathomechanism of the idiopathic rupture of calcaneal tendon it is necessary to describe the trajectory of the movement, than to calculate the path of the point of the initial insertion of the medial head of gastrocnemius right above the medial condyle of the femur from the position of the thigh flexed against the shank until the full extension of the knee. The knee is the largest articulation in the human body. It is formed by the femur, the tibia and also by a sesamoid bone - the patella (a kneecap). The knee articulates the thigh with the shank and thus it is also called the femoro-tibial joint. The articulation of the two bones within the knee occurs so that the two convex condyles of the femur, which

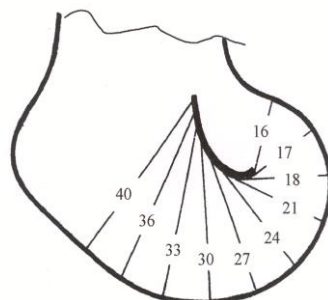


form the joint head, move on the concave surfaces of the condyles of the tibia which constitute the acetabulum. The flexion and extension of the knee is a combination of rolling and sliding [2,4]. During the first phases of the flexion, up to approx.  $20^\circ$ , the femur rolls on the tibia. Further rolling is, however, obstructed by the cruciate ligaments and the rolling turns into sliding, while other points on the condyles of the distal epiphysis of the femur are still in contact with the same points on the condyles of the proximal epiphysis of the tibia and the patella. The flexion range of the knee approaches  $\sim 130^\circ$  to  $150^\circ$  and is wider than that of any other joint in the body. The muscles are responsible for the flexion of up to  $\sim 130^\circ$  while the remaining part is caused by external forces. For the sake of facilitation and clarity of the kinematic description some simplifications were introduced to figure 1 in which the medial condyle of the femur is designated as "1" and the contour of the acetabulum of the tibia "0" is shown as a portion of a straight line. The point of the initial insertion of gastrocnemius on the femur "1" is marked as "M" and its trajectory – as  $\mu_M$ .



**Fig. 1**

Designations of the contour of the upper articular surface of the medial condyle of the tibia and the medial condyle of the femur as well as the trajectory of the point "M" of the initial insertion of the medial head of the gastrocnemius muscle

**Fig. 2**

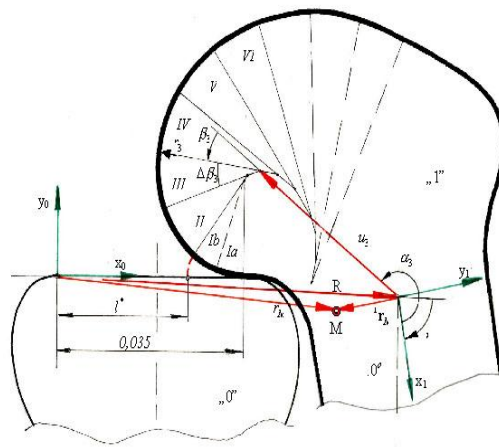
A sagittal cross-section of the medial condyle of the femur with the indicated radii of the curvatures

The markedly augmented distal end of the femur “1” is finished with two condyles: a bigger medial and smaller lateral ones which, during the flexion or extension of the knee, relocate against the upper surface of the medial condyle of the tibia “0”. Interaction of the two elements occurs along the contours  $C_1'C_1''$  -  $C_0'C_0''$  (Fig. 3) whereas  $C_1'C_1'' > C_0'C_0''$ ; hence, during the interaction, rolling must be accompanied by sliding. Consequently, the movement of the femur’s medial condyle against the upper surface of the juxtaposed condyle of the tibia is a complex process what adds to the difficulty in describing the movement which is necessary for estimation of the trajectory  $\mu_M$  of point “M”. In the description, the upper face of the medial condyle of the tibia is a flat surface while the route of the mutual interaction against the upper face of the tibia’s medial condyle  $C_0'C_0''$  is a straight line, and  $C_0'C_0''$  consists of the arcs forming the contour of the medial condyle of the femur (Fig. 2) [3].

The femur the tibia “0” were assigned the coordinate system  $x_0y_0$  and the femur “1” was assigned the coordinate system  $x_1y_1$ , whereas  $x_0y_0$  is a constant system (Fig 3). According to symbols shown in Fig. 3 the vector designating the position of point “M” within the base system  $x_0y_0$  can be described by the relationship (1):

$$(1) \quad r_M = R + A \cdot {}^1r_M$$

where:  ${}^1r_M$  defines the position of point “M” in the configuration of element “1”



**Fig. 3**  
Description of the initial state of the interaction

A is the matrix of transformation from the system  $x_1y_1$  to  $x_0y_0$  and takes the form of

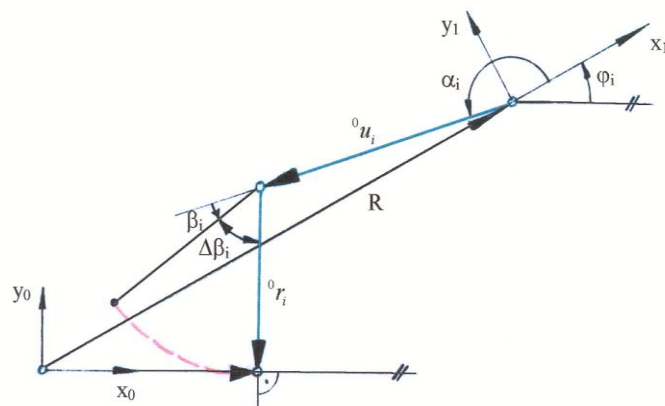
$$(2) \quad \mathbf{A} = \begin{bmatrix} \cos \varphi & -\sin \varphi \\ \sin \varphi & \cos \varphi \end{bmatrix}$$

where  $\varphi$  just as Fig. 3

In line with Fig. 4, which shows the  $i$ -th point of interaction, in each point of the interaction within the knee joint the following vector equation occurs:

$$(3) \quad \mathbf{R} + {}^0 \mathbf{u}_i + {}^0 \mathbf{r}_i - \mathbf{1}_i = 0$$





**Fig. 4**  
 Parameters describing the *i*-th point of interaction in the knee joint

Vectors  ${}^0\mathbf{u}_i$  and  ${}^0\mathbf{r}_i$  are obtained by transformation according to the following relationships:

$$(4) \quad {}^0\mathbf{u}_i = \mathbf{A} \begin{bmatrix} \cos \alpha_i \\ \sin \alpha_i \end{bmatrix} \mathbf{u}_i$$

$$(5) \quad {}^0\mathbf{r}_i = \mathbf{A} \begin{bmatrix} \cos(\alpha_i + \beta_i) \\ \sin(\alpha_i + \beta_i) \end{bmatrix} \mathbf{r}_i$$

where:  $\alpha_i$  *i*  $\beta_i$  just as Fig. 3

The vector describing the interaction point consists only of the component along the  $x_0$  axis

$$(6) \quad \mathbf{1}_i = [l_i \quad 0]^T$$

Using the equations (4), (5) and (6) equation (3) can be reduced to the following form:



$$(7) \quad \mathbf{R} = \begin{bmatrix} Rx \\ Ry \end{bmatrix} = \begin{bmatrix} l_i \\ 0 \end{bmatrix} - A \begin{bmatrix} u_i \cos \alpha_i + r_i \cos(\alpha_i + \beta_i) \\ u_i \sin \alpha_i + r_i \sin(\alpha_i + \beta_i) \end{bmatrix}$$

Considering the transformation matrix (2) one obtains

$$\begin{bmatrix} Rx \\ Ry \end{bmatrix} = \begin{bmatrix} l_i \\ 0 \end{bmatrix} - \begin{bmatrix} \cos \varphi & -\sin \varphi \\ \sin \varphi & \cos \varphi \end{bmatrix} \begin{bmatrix} u_i \cos \alpha_i + r_i \cos(\alpha_i + \beta_i) \\ u_i \sin \alpha_i + r_i \sin(\alpha_i + \beta_i) \end{bmatrix}$$

where:  $l_i$   $r_i$  just as Fig. 3

what, after doing the calculations, leads to relationships describing the position of the onset of the  $x_1y_1$  system in the following form:

$$(8) \quad Rx = l_i - u_i \cos \alpha_i \cos \varphi - r_i \cos(\alpha_i + \beta_i) \cos \varphi + u_i \sin \alpha_i \sin \varphi + r_i \sin(\alpha_i + \beta_i) \sin \varphi$$

$$(9) \quad Ry = -u_i \cos \alpha_i \sin \varphi - r_i \cos(\alpha_i + \beta_i) \sin \varphi - u_i \sin \alpha_i \cos \varphi - r_i \sin(\alpha_i + \beta_i) \cos \varphi$$

Relationships (8) and (9) along with (1) explicitly describe the position of point "M". Noteworthy, expressions (8) and (9) are the following functions:

$$R_x = R_x(l_i, \varphi, \beta_i, r_i, \alpha_i, u_i)$$

$$R_y = R_y(l_i, \varphi, \beta_i, r_i, \alpha_i, u_i)$$

while  $r_i$ ,  $\alpha_i$ ,  $u_i$  are constant parameters for one (i-th) segment of the contour  $C_1$   $C_1''$  and  $l_i$ ,  $\varphi$  and  $\beta_i$  are variables.

Assuming, in the first phase of the analysis, that the movement occurs without sliding, let's bring our attention to the fact that there is a relation between angles  $\alpha_i$ ,  $\varphi_i$  and  $\beta_i$ . As indicated in Fig. 6, for each point of the interaction a change of angle  $\beta_i$  within a single element, which is a sector of the circle, brings about a perpendicularity of vectors  $l_i$  and  $r_i$  and leads directly to the relationship:

$$(10) \quad \beta_i = 3\frac{\pi}{2} - \alpha_i - \varphi$$

With use of relationship (10), and the three variables in equations (8), (9) which were reduced to the two variables according to the expectations as indicated earlier,



during the flexion or extension of the knee the medial condyle of the femur "1" have two degrees of freedom (rolling and sliding) on the upper articular surface of the medial condyle of the tibia "0".

The two above degrees of freedom require variables  $l_i$ ,  $\varphi$  in relationships (8) and (9)

Difficulties in the description of the kinematics of the knee joint result from our lack of the quantitative evaluation of the slide. Indeed, only descriptive evidence of this phenomenon can be found in the literature [1, 2, 4]. It has been indicated that in the first phase of the extension only rolling takes place, which is followed by rolling and sliding until the full extension of the knee. As far as the extension angle at which the rolling occurs and from what position the sliding begins can be approximated, quantitative characteristics of the slide is virtually unknown. We do know, however, the total slide which can be estimated from the difference between the lengths of the interaction distances  $C_1'C_1''$  and  $C_0'C_0''$ . In this case, the relationship describing the position of the interaction point should be defined as:

$$(11) \quad l_i^*(\varphi) = l_i(\varphi) + p_i(\varphi)$$

The first element of this relationship expresses the effect of pure rolling and can be calculated from the relation:

$$l_i = l_{init} + \sum r_k \Delta \varphi_k$$

in which under the sum symbol is the relocation resulting from covering during the rolling of the consecutive segments of the femur's condyle ( $r_k \Delta \varphi_k$ ), as measured from the arbitrarily assumed initial value  $l_{init}$ . The second element of equation (11) describes the change in the slide expressed as the function of the angle of rotation of the femur. Because of the unknown character of this function, our own approach was proposed based on the frequently observed regularity in the living organisms that all changes in relocations, controlled by the autonomous nerve system, are generally carried out in a gentle manner. Hence, with reference to the kinematics parameters, one can assume that the sliding of the medial condyle of the femur obviously begins with the zero velocity and zero acceleration which then increase and, after attaining the extreme level, decrease and assume negative values (opposite to the velocity) when the motion approaches the end and the velocity drops to zero. In order to describe such a slide the sinusoid characteristics of the geometric acceleration of the slide [5] was assumed (Fig. 5), according to the equation:





$$(12) \quad \frac{d^2 p}{d\varphi} = B \sin \frac{2\pi}{\varphi_p} \varphi$$

Then, the geometric acceleration of the slide and the slide itself are expressed by the relationships:

$$(13) \quad \frac{dp}{d\varphi} = -B \left( \frac{\varphi_p}{2\pi} \right) \cos \frac{2\pi}{\varphi_p} \varphi + C_1$$

$$(14) \quad p = -B \left( \frac{\varphi_p}{2\pi} \right)^2 \sin \frac{2\pi}{\varphi_p} \varphi + C_1 \varphi + C_2$$

The B, C<sub>1</sub> and C<sub>2</sub> constants can be estimated using the boundary conditions employed for the above described character of the slide, i.e.:

$$\varphi = 0 \rightarrow \frac{dp}{d\varphi} = 0 \rightarrow C_1 = B \frac{\varphi_p}{2\pi}$$

$$\varphi = 0 \rightarrow p = 0 \rightarrow C_2 = 0$$

$$\varphi = \varphi_p \rightarrow p = s \rightarrow B = s \frac{2\pi}{\varphi_p^2}$$

Eventually, the slide can be expressed by the following relationship:

$$(15) \quad p = -\frac{s}{2\pi} \sin \frac{2\pi}{\varphi_p} \varphi + \frac{s}{\varphi_p} \varphi$$

where:

$\varphi_p$  - is the angle of rotation of the femur equivalent to the occurrence of the slide,  
 s - is the value of total slide taking place in the analyzed joint, and the values of the geometric velocity and acceleration are expressed by the equations:

$$(16) \quad \frac{dp}{d\varphi} = -\frac{s}{\varphi_p} \cos \frac{2\pi}{\varphi_p} \varphi + \frac{s}{\varphi_p}$$



$$(17) \quad \frac{d^2 p}{d\varphi^2} = -\frac{2\pi s}{\varphi_p^2} \sin \frac{2\pi}{\varphi_p} \varphi$$

Having estimated slide  $p$  from relationship (15) one can proceed to determining  $l_i^*(\varphi)$  and then to equations (8) and (9), and in the next step to estimation from relationship (1) of the consecutive points of the trajectory of point "M". The shape (character?) of this trajectory is now described by the geometric parameters and the variable angle  $\varphi$  of the rotation of the femur.

Actual dimensions of the medial condyle of the femur are presented in Fig. 2 and dimension of the remaining components of the knee are laid down in the Table below.

**Table 1**

Actual dimensions of the medial condyle of the femur [4] and of the remaining components of the knee joint (Fig. 3)

Segment no.	Motion type	r (m)	$\Delta\beta$ (deg)	$\beta$ (deg)	U (m)	$\alpha$ (deg)
Ia	rolling	0.016	20	106	0.036	222
Ib	rolling+sliding	0.016	18	88	0.036	222
II	rolling+sliding	0.017	31	57	0.036	220
III	rolling+sliding	0.018	25	35	0.035	217
IV	rolling+sliding	0.021	25	13	0.032	213
V	rolling+sliding	0.024	19	-6	0.028	211
VI	rolling+sliding	0.027	17	-9	0.023	213

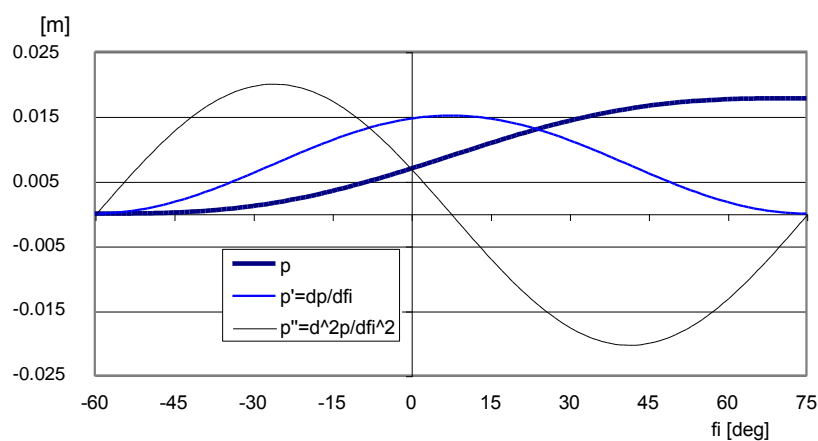
Based on the data from the above table the following values of the parameters necessary for determination of the shape of the trajectory of point "M" (Fig. 7) were estimated:

- assumed total rotation of the femur  $\varphi = 155^\circ$ ,
- first phase of the movement (segment Ia,  $20^\circ$ ) consists of pure rolling,
- second phase of the motion (segments Ib to VI,  $135^\circ$ ) consists of rolling + sliding,
- length of the medial condyle of the tibia (track C C): 0.035 m.,
- sum of the lengths of the arcs of the segments of the medial condyle of the femur (segments I to VI, C C): 0.0472 m.,
- length of the arc of segment Ia of the medial condyle of the femur (pure rolling): 0.0056 m.,



- total slide  $s$  (interaction on segments Ib to VI): 0.0178.

Using the above data and relationships (15), (16) and (17) values of the slide  $p$  corresponding to the variable angle  $\varphi$  were calculated and the results are presented in Fig. 5.



**Fig. 5**

The slide and its geometrical derivatives

( $p$  – slide,  $p'$  – velocity,  $p''$  – acceleration)

Based on the estimated values of coordinate  $l^*$  of the contact point, using the consecutive values of the  $\varphi$  angle, from relationships (8) and (9) one can determine coordinates of the beginning of the  $x_1y_1$  system, whereas relationship (1)

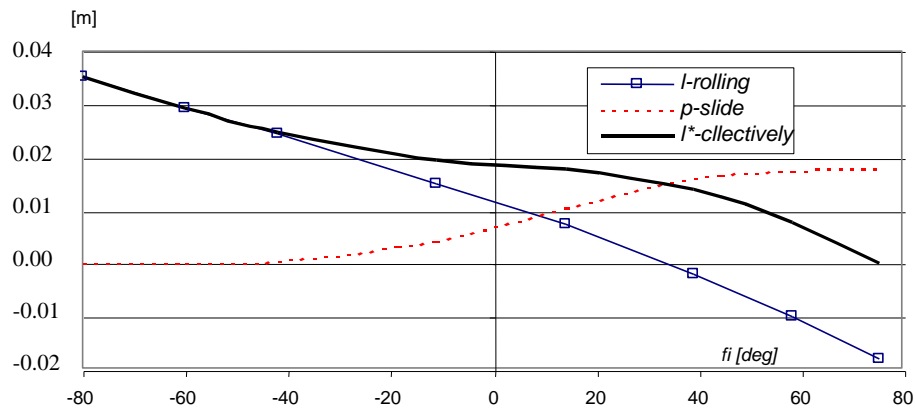
$\mathbf{I}_i = [0.01\text{m } 0]^T$  can be used to determine coordinates of the “M” point in the global  $x_0y_0$  system. Final results of the calculations presented as the trajectory of point “M” in the global system are shown in Fig. 7.

The estimated slide  $p$  allows now to define the coordinates of the point of contact of the condyle with the acetabulum of the tibia in the function of angle  $\varphi$  (Fig. 6).

1) can be used to determine coordinates of the “M” point in the global  $x_0y_0$  system.

Taking on the estimated values of coordinate  $l^*$  of the contact point, using the consecutive values of the  $\varphi$  angle, from relationships (8) and (9) one can determine coordinates of the beginning of the of the  $x_1y_1$  system whereas relationship (1) can be used to determine coordinates of the “M” point in the global  $x_0y_0$  system.

The vector describing point “M” in the local  $x_1y_1$  system was adopted from [1] as  $l_1 = [0.01 \text{ m } 0]^T$ . Final results of the calculations in the form of the trajectory of point “M” in the global system are presented in Fig. 7.



**Fig. 6**

Course of the value of the coordinate of the point of contact of the condyle with the acetabulum of the tibia:  $l$  for pure rolling,  $l^*$  for rolling and sliding – actual interaction

## Results

The trajectory (Fig. 7) was described by the nine-degree polynomial with the following coefficient values:

$$a_0 = 45.37775 \quad a_1 = 0.038235 \quad a_2 = 0.157479 \quad a_3 = -0.03727 \quad a_4 = 0.003823$$

$$a_5 = -0.00021 \quad a_6 = 7E-06 \quad a_7 = -1.3E-07 \quad a_8 = 1.36E-09 \quad a_9 = -5.8E-12$$



**Table 2**

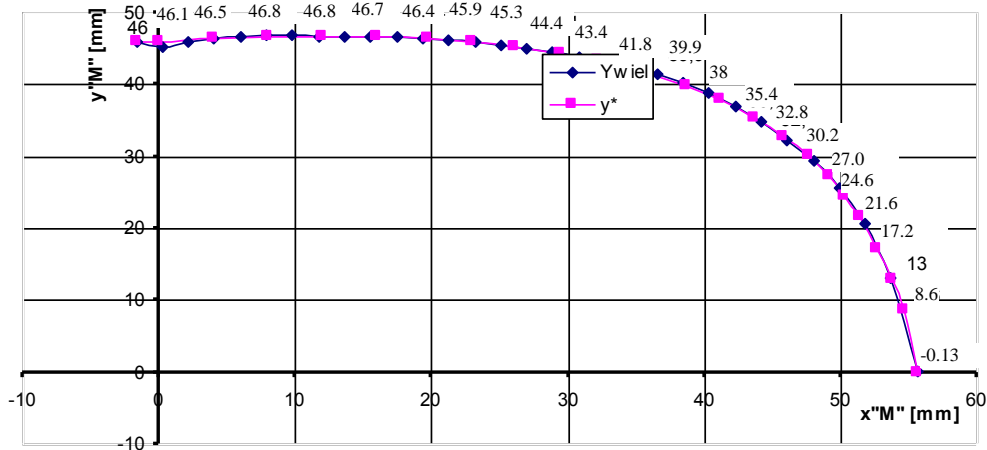
Coordinates of the points on the trajectory and the length of the route of the initial insertion "M" of the medial head of gastrocnemius from the flexion to the full extension of the knee

p no.	x	y	length increment	length from p3 in mm.	length from p4 in mm.
2	55.60	-0.13			
3	54.60	8.60	0.00	0.00	-16.59
3.1	53.70	13.00	4.49	4.49	-12.10
3.2	52.70	17.20	4.32	8.81	-7.78
3.3	51.40	21.60	4.59	13.40	-3.20
4	50.30	24.60	3.20	16.59	0.00
4.1	49.10	27.30	2.95	19.55	2.95
4.2	47.60	30.20	3.26	22.81	6.22
4.3	45.80	32.80	3.16	25.97	9.38
5	43.70	35.40	3.34	29.32	12.72
5.1	41.10	38.00	3.68	32.99	16.40
5.2	38.70	39.90	3.06	36.05	19.46
5.3	35.70	41.80	3.55	39.60	23.01
6	32.20	43.40	3.85	43.45	26.86
6.1	29.40	44.40	2.97	46.43	29.83
6.2	26.00	45.30	3.52	49.94	33.35
6.3	23.00	45.90	3.06	53.00	36.41
7	19.70	46.40	3.34	56.34	39.75
7.1	16.00	46.70	3.71	60.05	43.46
7.2	12.00	46.80	4.00	64.05	47.46
7.3	8.00	46.80	4.00	68.05	51.46
7.4	4.00	46.50	4.01	72.07	55.47
7.5	0.00	46.10	4.02	76.09	59.49
8	-1.70	46.00	1.70	77.79	61.20

The results presented in Table 2 demonstrate that the calculated maximal length of the route of point "M" – an initial insertion of the medial head of gastrocnemius – equals to 77.79 m. This route, however, is not real since it was calculated for the system in which certain simplification was used in the form of the flat upper articular surface of the medial condyle of the tibia. Despite this simplification, the



data collected in the present form and with the present precision allow to accurately describe the mechanics of the knee muscles.



**Fig. 7**

Trajectory of the “M” point of the initial insertion of the medial head of gastrocnemius on the femur for the extension of the knee joint.

## Discussion

Analysis of the kinematics of the knee joint from the flexion up to its full extension enabled the relatively precise definition and mathematical description (with the allowance for some simplification relating to the upper articular surface of the medial condyle of the tibia) of the significant parameters of the movement of the medial condyle of the femur along the upper articular surface of the medial condyle of the tibia. As a result, the size of the slide as well as its geometrical derivatives such as the vicinity and the acceleration could be estimated.

Based on the determined slide size coordinates of the points of contact of the condyle with the upper articular surface of the medial condyle of the tibia in the function of the  $\varphi$  angle were calculated. Utilizing the already estimated values of coordinates of the contact point  $l^*$  (for rolling and sliding) and assuming the consecutive values of the  $\varphi$  angle coordinates of the beginning of the  $x_1y_1$  system and then coordinates of the “M” point in the global  $x_0y_0$  system were determined. The calculated coordinates of the “M” point were described by the nine-degree

polynomial. By solving this polynomial the length of the route covered by the point of the initial insertion of the gastrocnemius muscle marked "M" could be calculated.

### Conclusions

The obtained results, along with the analysis of the mechanics of the muscles engaged in the movement of the human lower limb, the estimated forces generated by these muscles, the geometrical dimensions of the calcaneal tendon as well as the static attempt of the one-axial stretch of the calcaneal tendon to obtain data on the tendon's strength and elongation will provide basis for the definition of the idiopathic rupture of the calcaneal tendon (commonly called the Achilles tendon).

Computerized analysis of the shape of the upper surface of the medial condyle of the tibia and recording of this shape as a polynomial would allow to arrive at the least error-prone results.

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